Nonparametric Identification of Demand with Possibly Endogenous Characteristics

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  - → Derives novel moment conditions in a model closely related to Berry and Haile (2014)
  - → The rank condition for these moment conditions is non-standard relative to the NPIV literature—due to **asymmetry** of X and Z—which we call faithfulness

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   - → In Berry, Levinsohn, and Pakes (1995), suppose X does not have a RC, then

$$S_j(x, \tilde{x}, p; \theta) = \int \frac{\exp\left(\beta x_j + \xi_j + \alpha_i p_j + \gamma_i{}' \tilde{x}_j\right)}{\sum_k \exp\left(\beta x_k + \xi_k + \alpha_i p_k + \gamma_i{}' \tilde{x}_k\right)} dG_\theta(\alpha_i, \gamma_i)$$

   BLP inversion $\implies \delta_j := \beta x_j + \xi_j = \sigma_j^{-1}\left(S, P; \tilde{X}, \theta\right)$

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   (Aguirregabiria, Iaria, and Sokullu, 2023)

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   - → In Borusyak, Caceres-Bravo, and Hull (2025), Z is innovations in exchange rates, assumed exchangeable

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- Thus faithfulness packages (i) Z being a strong IV and (ii) X being a relevant proxy

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- Refine: for price counterfactuals, need exogenous variation in prices + proxies for δ

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  - → Exogeneity of X is an economically strong assumption $\rightsquigarrow$ turns out that's not needed!

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  - → Similar to Ahn (2025), who calls X's "prognostic variables"

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then completeness $\implies$ faithfulness

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(Price separability) If $P = f_0(f_1(X, Z) + f_2(X, \delta, \omega), \delta)$ then faithfulness $\iff$ completeness

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